

Set theory symbols:

\in (is a member of)

\cup (the union of)

\subseteq (is a subset of)

\emptyset (the empty/null set)

\cap (the intersection of)

\neq (is not equal to)

Categorial Grammar

Every	dog	ran
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(S/(S\NP))/N	N	S\NP
(e→t)→((e→t)→t)	e→t	e→t
λqλp.∀x[q(x)→(x)]	λx.dog'(x)	λx.ran'(x)

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S/(S\NP)
(e→t)→t
λp.∀x[dog'(x)→p(x)]

----- >

S
t
∀x.[dog'(x)→ran'(x)]

Set Theory Notation

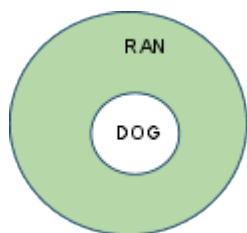
Every	dog	ran
λpλq. {x: p(x)} ⊆ {x: q(x)}	DOG	RAN

λpλq. {x: p(x)} ⊆ {x: q(x)}(DOG)
λq. {x: DOG(x)} ⊆ {x: q(x)}

λq. {x: DOG(x)} ⊆ {x: q(x)}(RAN)
{x: DOG(x)} ⊆ {x: RAN(x)}

In other words, the sentence "Every dog ran" is true iff every member of the set 'dog' is a member of the set 'things that ran'.

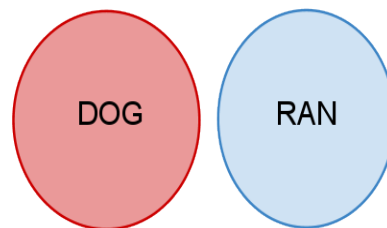
And the sentence "some dogs ran" is true iff the number of things from the set 'dog' that are also a member of the set 'things that ran' is greater than 0.



Every dog ran

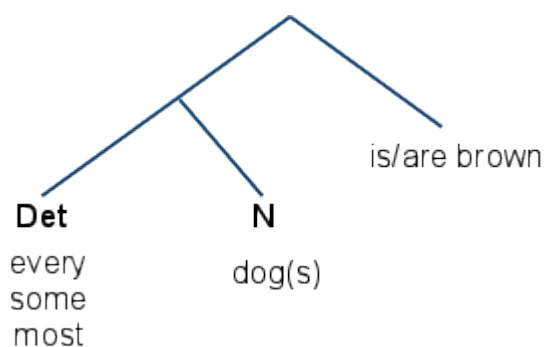


Some dogs ran



No dogs ran

Internal vs. External Arguments



In the case of determiners, they express a relationship between 2 sets. The determiner combines first with "dogs", and then combines with "are brown". Since the determiner combines lower in the syntactic hierarchy, we refer to this as the **internal** argument.

In the case of the sentence "Every dog is brown," you only need to consider the set denoted by the internal argument — DOGS. There is no need to look at any non-dogs to evaluate the sentence's truth. We say that the determiner "lives on" the internal argument, and the quantifier is **conservative**.

Any quantifier that fails to "live on" the internal argument is **nonconservative**. In other words, if to judge the truth or falsity of the sentence you have to look beyond the set denoted by the internal argument (DOGS, in this case), then the quantifier is nonconservative.

One of the biggest issues with our existing semantic theory (and, additionally, most current theories) is that it allows for nonconservative determiners, even though they are impossible in human languages. Ideally, we would need a theory that allows for only conservative determiners.

Set notation is also useful because it is impossible with lambda notation to derive the quantifier "most."